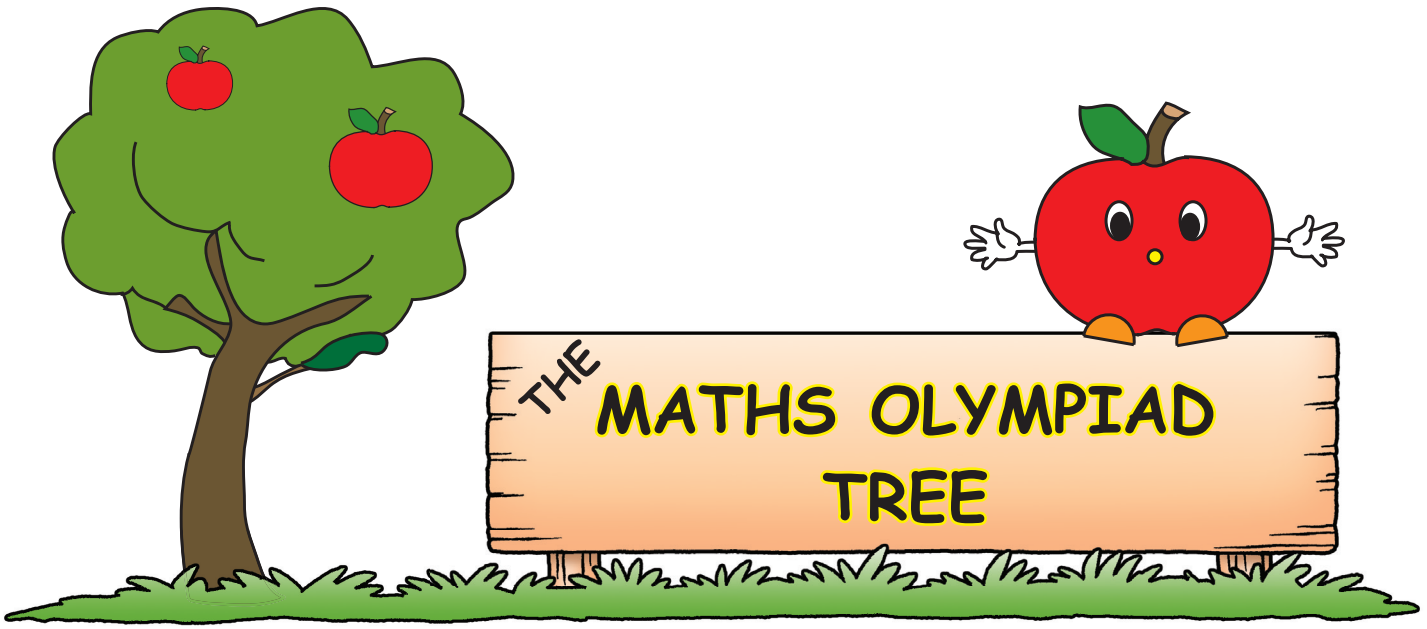
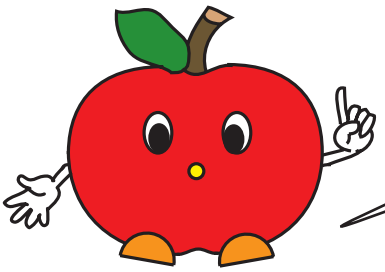


In this issue's "The Maths Olympiad Tree", join **Arithmetic Apple** on his journey to the Maths Olympiad Tree. We'll do a few questions on number series, frequently encountered in Maths competitions. Try these questions yourself first, then check against the solutions on the next page, to see if you got the answers right.



Question 1

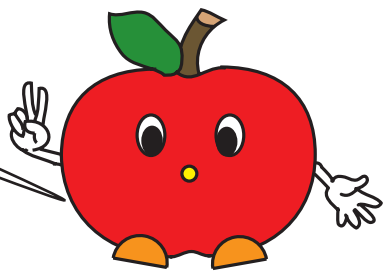
Evaluate the following number series:

$$\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\frac{1}{4\times5}+\ldots\ldots+\frac{1}{2008\times2009}+\frac{1}{2009\times2010}$$

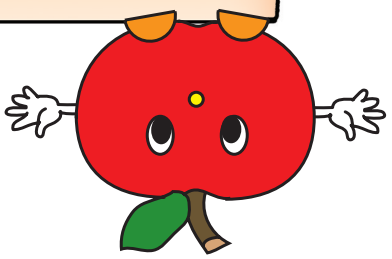
Question 2

Evaluate the following number series:

$$\frac{1}{1\times2}-\frac{1}{2\times3}-\frac{1}{3\times4}-\frac{1}{4\times5}-\ldots\ldots-\frac{1}{2008\times2009}-\frac{1}{2009\times2010}$$



SOLUTIONS!!!



Question 1

Method 1:

We analyse the long series from simpler and shorter cases, and see if we can spot any patterns.

$$\frac{1}{1\times2} = \frac{1}{2}$$

$$\frac{1}{1\times2} + \frac{1}{2\times3} = \frac{2}{3}$$

$$\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} = \frac{3}{4}$$

$$\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \frac{1}{4\times5} = \frac{4}{5}$$

Can you see the pattern now? The numerator is always 1 smaller than the denominator, which is the 'biggest number' we observe in the series.

Following the pattern, the answer must thus be $\frac{2009}{2010}$.

Method 2:

We first notice that every term in the series can be written as a difference of 2 unit fractions.

$$\frac{1}{1\times2} = \frac{1}{1} - \frac{1}{2} \qquad \frac{1}{2\times3} = \frac{1}{2} - \frac{1}{3}$$

$$\frac{1}{3\times4} = \frac{1}{3} - \frac{1}{4} \qquad \frac{1}{4\times5} = \frac{1}{4} - \frac{1}{5}$$

Hence, we can rewrite the entire sum of the series as:

$$\frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} +$$
$$\ldots\ldots \frac{1}{2008} - \frac{1}{2009} + \frac{1}{2009} - \frac{1}{2010}$$

Notice that terms between the first and last term all cancel out, so this leaves us with $1 - \frac{1}{2010} = \frac{2009}{2010}$.

Question 2

Method 1:

$$\frac{1}{1\times2} = \frac{1}{2}$$

$$\frac{1}{1\times2} - \frac{1}{2\times3} = \frac{1}{3}$$

$$\frac{1}{1\times2} - \frac{1}{2\times3} - \frac{1}{3\times4} = \frac{1}{4}$$

Following the pattern, the required answer is hence $\frac{1}{2010}$.

Method 2:

Let the answer to Question 1 be X and the answer to Question 2 be Y.

We notice that in X + Y, all the terms will cancel out except for the two $\frac{1}{1\times2}$, which gives a sum of 1.

Thus Y is just $1 - X = 1 - \frac{2009}{2010}$, which gives $\frac{1}{2010}$.